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Research Paper

Column Generation Model in Capacitated Multi-Periods Cutting Stock Problem with Pattern Set-Up Cost

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Abstract
Cutting Stock Problem (CSP) determines the cutting of stocks with standard length and width to meet the item's demand. The optimal solution will minimize the usage of stocks and trim loss. This research implemented the pattern generation algorithm for generating patterns. And then, we formulate the Gilmore-Gomory and Column Generation model in two-dimensional CSP. This CSP has two stages of cutting, whereas the first stage cut the stocks based on the width and the second stage based on the length. The Gilmore-Gomory model ensured that the first stage's strips were used in the second stage and met the item's demand. The Column Generation model added the pattern set-up cost as the constraint. The CSP in this research had three periods of cutting with different capacities in each period. The period is the unit of time for completing the demands. Based on the Column Generation model's solution, the first period used the 2^{nd}, 4^{th}, and 5^{th} patterns, the second period used the 4^{th} and 5^{th} patterns, and the third period did not use any patterns. The first and second periods fulfilled all of the demands.

Keywords
Cutting Stock Problem, Pattern Generation, Gilmore-Gomory, Column Generation

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1. INTRODUCTION

Industrial producers often face challenges in finding solutions for cutting raw materials. Raw materials are significant in production efficiency, so producers must optimize their usage. The problem of cutting raw materials is known as the Cutting Stock Problem (CSP). CSP is a problem of cutting raw materials (stocks) into small sizes (items) to minimize the remaining cutting.

The cutting problem divides the types into one dimension, two dimensions, and three dimensions based on the size. This study used two-dimensional CSP through a guillotine that cuts raw materials parallel. A set of raw materials with length and width of more than one size is called multiple stock sizes. Multiple stock sizes of two-dimensional CSP is one of the problems of cutting raw materials with more than one length and width. The remaining of cutting the raw material is called the trim loss. The formation of trim loss is due to inappropriate cutting patterns, so that the use of materials is excessive.

There has been a lot of researches on CSP, which started from one-dimensional CSP (Brandão et al. (2018); Arbib et al. (2016); Garraffa et al. (2016) ; Rodrigo and Shashikala (2017)). Rodrigo et al. (2012) proposed the pattern generation algorithm to determine patterns in two-dimensional CSP with location constraints. Then, Rodrigo et al. (2013) improved a method for solving the cutting of triangular shape items in two-dimensional CSP.

The researches continue to two-dimensional CSP. Andrade et al. (2016) formulated the model of two-staged two-dimensional CSP for stocks with useable leftover. Octarina et al. (2017) had designed a cutting pattern search application on the two-dimensional CSP. Furthermore, Octarina et al. (2018) and Octarina et al. (2019) implemented the Pattern Generation algorithm in forming the Gilmore-Gomory model. Pattern generation effectively generated the cutting patterns in two-dimensional CSP (Octarina et al., 2018).

Besides the algorithms for generating the patterns, the researchers also studied and developed the models of CSP. Octarina et al. (2019) formulated the Gilmore-Gomory model on a two-dimensional CSP for multiple stock sizes. The research showed that the Gilmore-Gomory model was useful for guillotine cutting type. Bangun et al. (2019) implemented the Branch and Cut method on the N-Sheet model. Some researchers also developed the researches for the irregular shaped item. Bangun et al. (2019) formulated the three-phase matheuristic model in two-dimensional CSP of triangular shape items. Octarina et al. (2020) implemented the modified branch and bound algorithm on the dotted board model for triangular shape items. Ma et al. (2018) developed a mixed-integer linear programming model and proposed an algorithm called dynamic programming-based on a heuristic to solve it. Ma et al. (2019) conducted a comparative analysis of models based on two-dimensional CSP with two-stage guillotine cutting, namely the Gilmore-Gomory models and the Arc flow model. According to Ma et al. (2019), the Gilmore-Gomory model is better than the Arc flow model for multiple cutting stock problems.

Column Generation Technique (CGT) can also solve two-dimensional CSP with guillotine type and fixed orientation (Etebari (2019); Lin and Hsu (2016)). Song and Bennell (2014) stated that Column Generation was a standard method for the CSP but can not solve optimally irregular shaped items. Furthermore, Octarina et al. (2019) applied CGT to two-dimensional CSP with various raw materials where the plates provided consisted of several stock sizes.

Based on this background, this research designed a cutting pattern for rectangular items with limited stocks. The search pattern used the PG algorithm. Then we formulated the patterns into the Gilmore-Gomory model and the Column Generation model by adding the pattern set-up cost as the constraint. The period of cutting with different capacity was also considered in the model. There have been limited studies concerned with multi-periods CSP. Therefore, this research intends to implement the column generation model incapacitated multi-periods CSP with pattern set-up cost. The LINGO 13.0 program completed the model.

2. EXPERIMENTAL SECTION

2.1 Data

This research used the data, which consisted of stock sizes and three different item sizes. The stock sizes are 3,000 mm × 3,000 mm respectively with the item sizes are 378 mm × 200 mm 555 mm × 496 mm, and 555 mm × 755 mm. For detail, it can be seen in Table 1.

Table 1. Size of items and demands

No	Size of items (mm)	Demands (pieces)
1	378 x 200	75
2	555 x 496	5
3	555 x 755	4

2.2 Methods

The steps taken in this study are:

a. Describe the data, including stock size (length and width) and the number of demand for each stock.

b. Implement the PG algorithm Octarina et al. (2018) to determine the cutting patterns and state the patterns into the table.

c. Formulate the Gilmore-Gomory model by defining the variables, objective function, and constraints. The variables defined the patterns, the objective function showed the minimum usage of stock, and the constraints ensured that the strips produced in the first stage would be used in the second stage. The constraints also stated that the optimal solution would fulfill all the demands of items.

d. Solve the Gilmore-Gomory model using the LINGO 13.0 application.

e. Formulate the Column Generation model and solve it by using the LINGO 13.0 application.

f. Analyze the final results.

3. RESULTS AND DISCUSSION

Table 1 shows that there were three types of items with different sizes and demand. The most number of demand was the first item with dimensions of 378 mm × 200 mm. There were two stages of cutting, whereas the early stage was cutting based on the width, and the second stage was based on the length. By using Pattern Generation (PG) algorithm Octarina et al. (2018) to the data in Table 1, there were 18 cutting patterns based on the width and 21 cutting patterns based on the length, which can be seen in Table 2 and Table 3 respectively.

Table 2. Cutting patterns according to the width

The j^{th} cutting pattern	The width of each item 755	496	200	Trim Loss
1	3	1	1	39
2	3	0	3	135
3	2	3	0	2
4	2	2	2	98
5	2	1	4	194
6	1	0	7	90
7	1	4	1	61
8	1	3	3	157
9	1	2	6	53
10	1	1	8	149
11	0	0	11	45
12	0	6	0	24
13	0	5	2	120
14	0	4	5	16
15	0	3	7	112
16	0	2	10	8
17	0	1	12	104
18	0	0	15	0

From Table 2 with the stock's width of 3,000 mm and by using the first pattern, there were three pieces of items with width 755 mm, an item with width 496 mm, an item with width 200 mm, and 39 mm of trim loss. The patterns continue until the 18^{th} pattern.

From Table 3 with the stock's length of 3000 mm and by using the first pattern, there were only five pieces of items with size 555 mm and 225 mm of trim loss, and so on until the 21^{st} pattern.

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Table 3. Cutting patterns according to the length

The j^{th} cutting pattern	The length of each item 555	555	378	Trim Loss
1	5	0	0	225
2	4	1	0	225
3	4	0	2	24
4	3	2	0	225
5	3	1	2	24
6	3	0	3	201
7	2	3	0	225
8	2	2	2	24
9	2	1	3	201
10	2	0	5	0
11	1	4	0	225
12	1	3	2	24
13	1	2	3	201
14	1	1	5	0
15	1	0	6	177
16	0	5	0	225
17	0	4	2	24
18	0	3	3	201
19	0	2	5	0
20	0	1	6	177
21	0	0	7	354

3.1 The Gilmore-Gomory Model

Then we implemented the cutting patterns from the Pattern Generation algorithm into the Gilmore-Gomory model, which aims to minimize the paper stock used in producing the ordered items.

Here are the steps for formulating the Gilmore-Gomory model (Octarina et al., 2018):

1. Defining Variables

The variables used in this study are as follows:

Z represents the objective function whose value is the minimum amount of paper stock to be cut.

J_0 represents the set of cutting patterns in the first stage.

l_i represents the length of ith item, i=1,2,3, so l_1 = 378 mm, l_2=555 mm, and l_3=555 mm.

w_i represents the width of i^{th} item, i = 1,2,3, so w_1 = 200 mm, w_2 = 496 mm, and w_3 = 755 mm.

λ_j^0 represents the number of stocks which were cut in the first stage based on the j^{th} pattern.

λ_j^s represents the number of stripe with length l_s and width W, s ∈ {1,2,…,m'} which were cut in the second stage based on the j^{th} pattern.

b_i represents the demand of i^{th} item.

M_s is the sth sub-matrix, where s ∈ {1,2,…,m'} and the set of the patterns in the second stage, which produce item with length $l_i \leq l_s$. For example the 2^{nd} sub-matrix is the set of patterns which makes the item with a dimension of 555 mm and 378 mm.

M_0 is the sub-matrix which consist of the set of patterns in the first stage. The entry of sub-matrix represents the number of item with length l_i which was cut according to the jth pattern.

$$M' = \begin{bmatrix} 1 & 3 & 0 & 2 & 4 & 7 & 1 & 3 & 6 & 8 & 11 & 0 & 2 & 5 & 7 & 10 & 12 & 15 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 3 & 2 & 1 & 0 & 4 & 3 & 2 & 1 & 0 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 3 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix}$$

$$M'' = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 6 & 5 & 3 & 2 & 0 & 6 & 5 & 3 & 2 & 0 & 5 & 3 & 2 & 0 & 3 & 2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 & 5 & 0 & 1 & 2 & 3 & 4 & 0 & 1 & 2 & 3 & 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 3 & 3 & 3 & 4 & 4 & 5 \end{bmatrix}$$

$$M = \left[\begin{array}{c|ccc|ccc|c|ccc} & -1 & \ldots & -1 & & 0 & & & & 0 & \\ & & 0 & & -1 & \ldots & -1 & & & \vdots & \\ M_0 & & \vdots & & & 0 & & \ldots & & & \\ & & & & & \vdots & & & & 0 & \\ & & 0 & & & 0 & & & -1 & \ldots & -1 \\ \hline 0 & & M_1 & & & M_2 & & \ldots & & M_{m'} & \end{array}\right]$$

M' and M'' represent m' first row and m last row of matrix M respectively.

The objective function in this problem is to minimize the amount of stock but still meet the demand. The Gilmore-Gomory model can be seen in Model (1)-(4).

Minimize

$$Z = \sum_{j=1}^{18} \lambda_j^0 \tag{1}$$

Subject to

$$M'\lambda = 0 \tag{2}$$

$$M''\lambda \geq b \tag{3}$$

$$\lambda \geq 0 and integer \tag{4}$$

with

Constraint (2) ensured that all stripes in the first stage were used in the second stage and cut into the requested items. Constraint (3) confirmed that the demand for items was fulfilled. We write the Constraint (2) and Constraint (3) into an M matrix.

The cutting patterns from Table 2 were sorted according to the width in ascending order and formed M'. Simultaneously, the cutting patterns from Table 3 were also sorted according to the length in ascending order. The value of λ was defined as follows.

$$\lambda = [\lambda_1^0 ... \lambda_j^0, \lambda_1^1 ... \lambda_j^1, \lambda_1^2 ... \lambda_j^2 ... \lambda_1^{m'} ... \lambda_j^{m'}]^T$$

According to Model (1)-(4), the Gilmore-Gomory model can be stated as Model (5-12).

Minimize

$$\begin{aligned} z = \; & \lambda_1^0 + \lambda_2^0 + \lambda_3^0 + \lambda_4^0 + \lambda_5^0 + \lambda_6^0 + \lambda_7^0 + \lambda_8^0 + \lambda_9^0 + \lambda_{10}^0 \\ & + \lambda_{11}^0 + \lambda_{12}^0 + \lambda_{13}^0 + \lambda_{14}^0 + \lambda_{15}^0 + \lambda_{16}^0 + \lambda_{17}^0 + \lambda_{18}^0 \end{aligned} \tag{5}$$

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$$\lambda_1^0 + 3\lambda_2^0 + 2\lambda_4^0 + 4\lambda_5^0 + 7\lambda_6^0 + \lambda_7^0 + 3\lambda_8^0 + 6\lambda_9^0 + 8\lambda_{10}^0 + 11\lambda_{11}^0 + 2\lambda_{13}^0 + 5\lambda_{14}^0 + 7\lambda_{15}^0 + 10\lambda_{16}^0 + 12\lambda_{17}^0 + 15\lambda_{18}^0 - \lambda_1^1 = 0 \quad (6)$$

$$\lambda_1^0 + 3\lambda_3^0 + 2\lambda_4^0 + \lambda_5^0 + 4\lambda_7^0 + 3\lambda_8^0 + 2\lambda_9^0 + \lambda_{10}^0 + 6\lambda_{12}^0 + 5\lambda_{13}^0 + 4\lambda_{14}^0 + 3\lambda_{15}^0 + 2\lambda_{16}^0 + \lambda_{17}^0 - \lambda_1^2 - \lambda_2^2 - \lambda_3^2 - \lambda_4^2 - \lambda_5^2 = 0 \quad (7)$$

$$3\lambda_1^0 + 3\lambda_2^0 + 2\lambda_3^0 + 2\lambda_4^0 + 2\lambda_5^0 + 2\lambda_6^0 + \lambda_7^0 + \lambda_8^0 + \lambda_9^0 + \lambda_{10}^0 + \lambda_{11}^0 - \lambda_1^3 - \lambda_2^3 - \lambda_3^3 - \lambda_4^3 - \lambda_5^3 - \lambda_6^3 - \lambda_7^3 - \lambda_8^3 - \lambda_9^3 - \lambda_9^3 - \lambda_{10}^3 - \lambda_{11}^3 - \lambda_{12}^3 - \lambda_{13}^3 - \lambda_{14}^3 - \lambda_{15}^3 = 0 \quad (8)$$

$$7\lambda_1^1 + 6\lambda_1^2 + 5\lambda_2^2 + 3\lambda_3^2 + 2\lambda_4^2 + 6\lambda_1^3 + 5\lambda_2^3 + 3\lambda_3^3 + 2\lambda_4^3 + 5\lambda_5^3 + 3\lambda_6^3 + 2\lambda_7^3 + 3\lambda_8^3 + 2\lambda_9^3 + 2\lambda_{10}^3 \geq 75 \quad (9)$$

$$\lambda_1^2 + 2\lambda_2^2 + 3\lambda_3^2 + 4\lambda_4^2 + 5\lambda_5^2 + \lambda_1^3 + 2\lambda_2^3 + 3\lambda_3^3 + \lambda_4^3 + 2\lambda_5^3 + \lambda_6^3 \geq 6 \quad (10)$$

$$\lambda_1^3 + \lambda_2^3 + \lambda_3^3 + \lambda_4^3 + \lambda_5^3 + 2\lambda_6^3 + 2\lambda_7^3 + 2\lambda_8^3 + 2\lambda_9^3 + 3\lambda_{10}^3 + 3\lambda_{11}^3 + 3\lambda_{12}^3 + 4\lambda_{13}^3 + 4\lambda_{14}^3 + 5\lambda_{15}^3 \geq 4 \quad (11)$$

$\lambda \geq 0$ and integer

$$\lambda = [\lambda_1^0 ... \lambda_j^0, \lambda_1^1 ... \lambda_j^1, \lambda_1^2 ... \lambda_j^2 ... \lambda_1^{m'} ... \lambda_j^{m'}]^T \quad (12)$$

Constraint (6)-(8) ensured that the strip with a width of 200 mm, 496 mm, and 755 mm, produced in the first stage, will be used in the second stage. Constraint (9)-(11) ensured that the demand for each items was fulfilled. Constraint (11) showed the non-negative and integer solution. By using the LINGO 13.0, the optimal solutions of Model (5-12) were

$$Z = 2; \lambda_6^0 = 1; \lambda_{16}^0 = 1; \lambda_1^1 = 17; \lambda_4^2 = 2; \lambda_8^3 = 1; \lambda_{15}^3 = 1$$

The optimal solutions which equal to one means that we used the 6ˆth and 16ˆth cutting patterns in the first stage. For the second stage, we used:

1. The 1^{st} cutting pattern on the stripe with a length of 378 mm,
2. The 4^{th} cutting pattern on the stripe with a length of 555 mm, and
3. The 8^{th} and 15^{th} cutting patterns on the stripes with a length of 555 mm.

The 6^{th} cutting pattern corresponding to the width, produced an item of 755 mm and seven items of 200 mm. The 16^{th} cutting pattern produced two items of 496 mm and ten items of 200 mm. The stripe with length 378 mm was used for the 1^{st} cutting pattern and produced five items of 555 mm, while the stripes with length 555 mm was used for the 4^{th}, 8^{th} and 15^{th} cutting patterns. The 4^{th} cutting pattern yields five items of 555 mm, the 8^{th} cutting pattern yields four items of 555 mm and two items of 378 mm. The 15^{th} cutting pattern produced an item of 555 mm and six items of 378 mm.

3.2 The Column Generation Model

Column Generation model aims to minimize trim loss and costs. If the cost reduction is in a negative value, the solution can be entered as a new column. If the cost reduction is more significant than or equal to zero, a lower bound for the optimal solution has been found, although not an integer solution. According to Ma et al. (2019), they used round up, round down, or combined rounds to generate an integer solution in solving a single CSP period. Given each period's production capacity constraints, round it up to non-integer components and then solve the residual problem. We formulated the Column Generation model for this problem into Model (13)-(17).

Minimize

$$Z = \sum_{t=1}^{T} \left\{ \sum_{j} \left\{ (C + \sum_{i=1}^{n} A_{it} a'_{ijt}) y_{jt} + \beta \right\} z_t^j \right\} \quad (13)$$

Subject to:

$$\sum_{t=1}^{T} \sum_{j} y_{jt} a'_{ijt} z_t^j \geq \sum_{t=1}^{T} d_{it} \forall i, t \quad (14)$$

$$\sum_{j} y_{jt} z_t^j \geq Qt \forall t \quad (15)$$

$$z_t^j \in \{0, 1\} \forall j, t \quad (16)$$

$$A_{it} = h_i (T + 1 - t) \quad (17)$$

Constraint (14) showed that all the demands were fulfilled. Constraint (15) showed the production capacity and Constraint (16) showed that the decision varibles were 0 or 1.

where n is the number of item, n=3

T is the number of period, $T = 3$

$d_i t$ is the demand of the i^{th} item in the t^{th} period

h_i is the inventory cost per unit per period of the i^{th} item, $h_i = 0.01\ l_i$

Q_t is the production capacity of the t^{th} period, $Q_t = 2\ d_{it}$

C is a unit cost, C = L

β is the pattern set-up cost, β = 0,01 L

z_t^j is the decision variable, the j^{th} pattern which cut from the t^{th} period

a'$_{ijt}$ is the number of the i^{th} item, which will cut according to the j^{th} pattern from the t^{th} period

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y_{jt} is the number of the j^{th} pattern from the t^{th} period
In details, the Model (13)-(17) can be stated in Model (18)-(23).
Minimize

$$\begin{aligned} z = {} & 12493320z_1^1 + 12493320z_2^1 + 12493320z_3^1 + 12493320z_1^2 + \\ & 12493320z_2^2 + 12493320z_3^2 + 13742520z_1^3 + 13742520z_2^3 + \\ & 13742520z_3^3 + 2498640z_1^4 + 2498640z_2^4 + 2498640z_3^4 + \\ & 12493320z_1^5 + 12493320z_2^5 + 12493320z_3^5 + 12493320z_1^6 \\ & + 12493320z_2^6 + 12493320z_3^6 \end{aligned} \tag{18}$$

Subject to

$$\begin{aligned} & 3z_1^1 + 165z_3^1 + 18z_1^2 + 6z_2^2 + 3z_3^2 + 6z_2^3 + 12z_1^4 + 6z_2^4 \\ & +21z_1^5 + 30z_2^5 + 6z_2^6 + 18z_3^6 \geq 85 \end{aligned} \tag{19}$$

$$\begin{aligned} & z_1^1 + z_2^1 + z_3^1 + z_1^2 + z_2^2 + z_3^2 + 11z_1^3 + 11z_2^3 + 11z_3^3 + 2z_1^4 + \\ & 2z_2^4 + 2z_3^4 + z_1^5 + z_2^5 + z_3^5 + z_1^6 + z_2^6 + z_3^6 \leq 150 \end{aligned} \tag{20}$$

$$\begin{aligned} & z_1^1 + z_2^1 + z_3^1 + z_1^2 + z_2^2 + z_3^2 + 11z_1^3 + 11z_2^3 + 11z_3^3 + 2z_1^4 + \\ & 2z_2^4 + 2z_3^4 + z_1^5 + z_2^5 + z_3^5 + z_1^6 + z_2^6 + z_3^6 \leq 12 \end{aligned} \tag{21}$$

$$\begin{aligned} & z_1^1 + z_2^1 + z_3^1 + z_1^2 + z_2^2 + z_3^2 + 11z_1^3 + 11z_2^3 + 11z_3^3 + 2z_1^4 + \\ & 2z_2^4 + 2z_3^4 + z_1^5 + z_2^5 + z_3^5 + z_1^6 + z_2^6 + z_3^6 \leq 8 \end{aligned} \tag{22}$$

$$z_t^j \in \{0, 1\}, j = 1, 2, 3, 4, 5, 6 and t = 1, 2, 3 \tag{23}$$

By using the LINGO 13.0, the solution of Model (18)-(23) were

$$\begin{aligned} & z = 0.34; z_1^1 = 0; z_2^1 = 0; z_3^1 = 0; z_1^2 = 1; z_2^2 = 0; z_3^2 \\ & = 0; z_1^3 = 0; z_2^3 = 0; z_3^3 = 0; z_1^4 = 1; z_2^4 = 1; z_3^4 = 0; \\ & z_1^5 = 1; z_2^5 = 1; z_3^5 = 0; z_1^6 = 0; z_2^6 = 0; z_3^6 = 0 \end{aligned}$$

The value of λ_j^s which equals to one means that the 2^{nd}, 4^{th}, and 5^{th} patterns were used in the 1^{st} period, the 4^{th} and 5^{th} patterns were used in the 2^{nd} period and the 3^{rd} period did not use anything, which means that the demand was fulfilled in the 1^{st} and 2^{nd} period. The Gilmore-Gomory model did not include a cutting period in the model, while the Column Generation model had it. The Column Generation model is used to minimize costs.

4. CONCLUSIONS

From the result and discussion, it can be concluded that the Gilmore-Gomory model and the Column-Generation model can be implemented in the Cutting Stock Problem, especially in multi-period CSP. The objective function of the Gilmore-Gomory model is to minimize the amount of stock but can meet the demand for each item. The Gilmore-Gomory's model constraints ensured that each strip produced in the first cutting stage could be used in the second stage, and the constraints provided all requests of the items. At the same time, the objective function of the Column Generation model is to minimize the cost. Adding the period constraint to the Column Generation model, this case's objective function is smaller than the Gilmore-Gomory model. We can see that the value of z in the Column Generation model is smaller than in the Gilmore-Gomory model.

For further research, the Cutting Stock Problem model's more extensions are critically essential to improve than previous models. We suggest computational tests for further study.

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